The problem:

Given a grid and pieces, find the sum of the costs of all possible arrangements of the pieces on the grid. The k pieces must be on distinct squares on the grid.

Try to think of a solution before reading below.

--------------------------------------------------------------------------------------------------------------------------------------

First, let’s understand the problem. If you have not noticed yet, it’s basically asking for the sum of the Manhattan / taxicab distance between each pair of pieces for all arrangements of the k pieces on the grid.

Obvious observation: you can calculate x and y independently

Now, let’s learn a new technique to solve this kind of problem: contribution technique. Instead of brute forcing every arrangement, we can count the ‘contribution’ of each component in the answer.

Let’s just look at the x component, and let’s set to be 1. How many arrangements will have 1 for ? Well, you need to choose 2 adjacent layers, and then choose 1 square from each layer. Let’s denote as the number of arrangements that has for .

means there are ways to choose the adjacent layers, is the number of possible ways to choose a square from each of the 2 layers, which each have squares, and the last term is the combination of ways to choose the positions of the rest of the pieces.

Now, what about in general? It’s actually pretty simple.

If becomes 2 (), there will be 1 less way to choose the 2 layers than in the case of .

What is the sum of all for every arrangement?

It’s just the sum of all times the number of arrangements that contains , or the sum of every times the ‘contribution’ of !

Therefore, the answer for x component is .

Once you know how to calculate for x, you can just swap and to calculate the answer for y.

Implementation:

You need to calculate

Now, you can just pull out some factors

It is pretty easy to calculate, except the combination part. How do you divide something in modulus?

First, you need to know what modular inverse is. (if you don’t just google it)

Fermat’s little theorem states that if is prime, for any . (it’s actually a bit more complicated than that but you can just use this version now) This can be easily modified into

Therefore, you can just use modular exponentiation (big mod) to calculate to calculate the modular inverse of .

--------------------------------------------------------------------------------------------------------------------------------------

These 2 tricks contribution technique and Fermat’s little theorem are used in almost all combinatorics problem. Contribution technique is a powerful tool to solve most problems involving counting stuff. Instead of exhausting all combinations, you calculate the ‘contribution’ of each piece of the answer, and sum it up. You are essentially looking at the problem from a completely different angle, and it is a basic but at the same time advanced skill that may even be applied in HKOI junior problems. This problem is a good introduction to these skills, so please do it.

<https://atcoder.jp/contests/abc127/tasks/abc127_e>